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Title: Feedback stabilization: a rich interplay between algebra and analysis.

Abstract: These lectures aim to illustrate the following comment by R. E. Kalman, P. L. Falb, and M. A. Arbib regarding the mathematical theory of systems:

“We have also tried to convey the fundamental notion that system theory is not simply a branch of applied analysis, but provides a source of problems and intuition for a rich interplay between algebra and analysis.”

This comment appears in the preface to their book *Topics in Mathematical System Theory*, published by McGraw-Hill in 1969.

We will demonstrate that mathematical systems theory is deeply connected not only with applied analysis but also with core mathematical disciplines such as algebra, analysis, geometry, and number theory.

To achieve this, we will focus on input-output stabilisation problems for linear systems defined in the frequency domain and consider the following classes of systems:

- finite-dimensional systems (ordinary differential and recurrence equations);
- infinite-dimensional systems (time-delay and partial differential equations);
- multidimensional systems (n D-filters, time-delay equations).

First, we will recast the fractional representation approach to analysis and synthesis problems developed by Vidyasagar, Desoer, Francis et al. (Desoer, Liu, Murray, and Saeks; Vidyasagar, Schneider, and Francis; Vidyasagar; Logemann; Curtain and Zwart), which lies at the heart of the development of robust control (H_∞ methods) (Zhou, Doyle, and Glover; Vinnicombe; Vidyasagar), in a modern algebraic analysis framework incorporating modules, fractional ideals, and lattices over normed rings and Banach algebras (Q.03a, Q.03b, Q.03c, Q.04, Q.06a, Q.06b, Q.25).

In this framework, we will characterize various concepts of stabilizability, such as internal, strong, simultaneous, and robust stabilizability, in terms of algebraic structures (Q.03a, Q.03b, Q.03c, Q.04, Q.05, Q.06a, Q.06b, Q.25, Sasane). In particular, we will demonstrate that the main concept of feedback stabilization (internal stabilization) is equivalent to a mathematical concept that plays a key role in modern algebra (module theory and homological algebra), as well as in differential topology and algebraic, differential, and non-commutative geometry.

Finally, we will also demonstrate how symbolic and symbolic-numeric methods of modern computer algebra can effectively address some of these issues in specific classes of linear systems.

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