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Title: The control of PDEs: some basic concepts, recent results and open problems.

Abstract: These Notes deal with the control of systems governed by some PDEs. I will mainly consider time-dependent problems. The aim is to present some fundamental results, some applications and some open problems related to the optimal control and the controllability properties of these systems. First, I will review part of the existing theory for the optimal control of partial differential systems. This is a very broad subject and there have been so many contributions in this field over the last years that we will have to limit considerably the scope. In fact, I will only analyze a few questions concerning some very particular PDEs. Then, I will recall results concerning the controllability of systems governed by linear time-dependent PDEs. Mainly, we will deal with the heat and the wave equations. I will try to explain which is the meaning of controllability and which kind of controllability properties can be expected to be satisfied by each of these PDEs. The main related results, together with the main ideas in their proofs, will be recalled. This will be followed by the presentation of several controllability results for other time-dependent, mainly nonlinear, parabolic systems of PDEs: first, the heat equation and some extensions; then, stochastic PDEs; also, some nonlinear systems from fluid mechanics: Burgers, Navier-Stokes, Boussinesq, micropolar, etc. Along these lectures, a set of questions (some of them easy, some of them more intricate or even difficult) will be stated. Moreover, several open problems will be mentioned. I hope that all this will help to understand the underlying basic concepts and results and to motivate research on the subject.